Cracking Right-Angled Triangles with Pythagoras and Ratios

Teacher		**Class**	Year 9
Subject	Maths	**Topic**	Pythagoras' theorem
Date		**Room**	
Lesson length	60 minutes	**Country**	United States

Students learn to use Pythagoras and matching ratios in similar triangles. They sort triangle families and solve missing-side problems.

Curriculum links

- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- Use Trigonometry to find a missing length/angle [1]
- Solving Problems involving Trigonometry [1]

Learning objectives

- Students will use Pythagoras' theorem to find missing sides in right-angled triangles.
- Students will use scale factor and trig ratios in similar triangles to find unknown sides.

Success criteria

- I can find the hypotenuse and short sides using $a^2 + b^2 = c^2$.
- I can use matching ratios in similar triangles to calculate missing sides.

What students already know

2-minute quick-fire: ask 3^2, 4^2, 6^2, $\sqrt{25}$, $\sqrt{144}$, and point to hypotenuse on a sketch.

Key words

hypotenuse	longest side, opposite the right angle in a right-angled triangle [3]
similar	same shape with same angles and sides in proportion [12]
perpendicular	meets at 90 degrees, like a wall meets a floor
scale factor	multiplier that enlarges one shape to match another of the same shape
ratio	comparison of two amounts found by division, kept the same in similar triangles [12]

Materials

- Slides 1-12 projected
- Set of 12 right-angled triangle cards in 4 families of 3 sizes, right angle marked
- Squared paper
- Calculators
- Rulers
- Mini whiteboards
- Worksheet Task D copies

Before the lesson

- Print and cut one set of 12 triangle cards per pair, with right angle and acute angles marked
- Print Worksheet Task D, one copy per student
- Test Slide 6 Video clip 1 (0:00–4:45) and Slide 8 Video clip 2 (0:00–5:00) with sound
- Set out calculators, squared paper, rulers and mini whiteboards

Created with LessonDino.com · AI lesson planner with checkable sources

Lesson outline

0–7 min **Starter** 7 min · Whole class

Recall squares and spot hidden right-angled triangles to launch both tools.

YOU DO

1. Display Slide 1 and give the hook for the main card sort.
2. Show Slide 2 mystery: a ladder is 3 m out, 4 m up, how long is it.
3. Quick-fire 3^2, 4^2, $\sqrt{25}$ and ask students to point to the hypotenuse.
4. Ask pairs to predict the ladder length and what maths would prove it.

STUDENTS DO

1. Call out squares and square roots on mini whiteboards.
2. Discuss with a partner where the right angle and hypotenuse are in the ladder sketch.
3. Write a prediction of the ladder length.

Say: These triangles are all different sizes but hiding the same secret — find the pairs. Look at this ladder: 3 m out on the ground, 4 m up the wall. It hides a right-angled triangle, and today we will crack it with Pythagoras and matching ratios.

Worked example: Ladder legs 3 m and 4 m, hypotenuse c: $3^2 + 4^2 = 9 + 16 = 25 = c^2$, so $c = \sqrt{25} = 5$ m.

Ask: Where is the right angle and which side is the hypotenuse? (Right angle between ground and wall; hypotenuse is the ladder, the longest side opposite the right angle [3].) · What is $3^2 + 4^2$ and what does it tell us? ($9 + 16 = 25$, which is 5^2, so the hypotenuse is 5 m.)

Check understanding: All show prediction on whiteboards; if squares are shaky, practise 5^2, 6^2, 12^2 and $\sqrt{36}$, $\sqrt{169}$ before moving on.

Support: Give a square-number grid to 15^2 for quick look-up during questioning. **Stretch:** Ask what 6 m out and 8 m up would give and why the answer doubles to 10 m.

Use: Slides 1-2

7–20 min **Teaching** 13 min · Whole class

Model finding the hypotenuse and a short side using $a^2 + b^2 = c^2$.

YOU DO

1. Show Slide 3 and name the right angle and hypotenuse as longest side opposite it [3].
2. Show Slide 4 cards: add squares then square root for hypotenuse; subtract then square root for a short side.
3. Play Slide 6 Video clip 1 (0:00–4:45) and tell students to spot when to add or subtract.
4. Model two examples on the board with full substitution and estimate check.

STUDENTS DO

1. Label a, b, c on their sketch and copy the two worked lines.
2. Tell a partner when to add and when to subtract the squares.
3. Estimate answers before calculating to check size sense.

Say: For any right-angled triangle, $a^2 + b^2 = c^2$ always works here [3]. The letter c is the hypotenuse, the longest side opposite the right angle [3]. If you want the hypotenuse you add the squares, if you want a short side you subtract.

Worked example: Hypotenuse with legs 6 cm and 8 cm: $6^2 + 8^2 = 36 + 64 = 100 = c^2$, so $c = \sqrt{100} = 10$ cm. Short side with hypotenuse 13 cm and leg 5 cm: $13^2 - 5^2 = 169 - 25 = 144$, so missing side $= \sqrt{144} = 12$ cm.

Ask: Why do we do $6^2 + 8^2 = c^2$ here and not a subtraction? (Because we want the hypotenuse, so sum of squares on the two shorter sides equals square on hypotenuse [4].) · With hypotenuse 13 cm and one leg 5 cm, what first line shows the subtraction? (Other leg$^2 = 13^2 - 5^2 = 169 - 25 = 144$, so side = 12 cm.) · How do you know 10 cm is sensible for legs 6 cm and 8 cm? (It is longer than 8 cm but shorter than 6 + 8, so 10 fits.)

Check understanding: Hinge with whiteboards: legs 5 cm and 12 cm, hypotenuse? If many miss 13 cm, re-model adding step.

Support: Provide a desk card showing steps: square both known sides, add or subtract, square root, with hypotenuse marked as longest side [3]. **Stretch:** Include decimal legs 2.1 cm and 2 cm where hypotenuse is 2.9 cm, to 1 decimal place.

Use: Slides 3-4, 6, Video clip 1 (0:00–4:45)

20–32 min **Guided practice** 12 min · Whole class

Link similar right-angled triangles to equal side ratios for finding sides.

YOU DO

1. Pose Slide 5 question: two triangles with 90°, 37°, 53°, what will match.
2. Build Slide 7 mindmap together: same angles, sides in same proportion, ratio finds missing sides.
3. Play Slide 8 Video clip 2 (0:00–5:00) to show how similar triangles give same ratios and how to pick matching sides.
4. Model one ratio calculation with opposite divided by hypotenuse to 2 decimal places.

STUDENTS DO

1. Call out what matches between the 37° twins: angles and side ratios.
2. Copy the mindmap and underline hypotenuse = longest and similar = same angles.
3. Calculate 3 ÷ 5 and 6 ÷ 10 on calculators and compare.

Say: Similar triangles have the same shape, same angles and sides in proportion. That gives the three trigonometric ratios in similar right-angled triangles [12]. If the angles match, opposite divided by hypotenuse will be nearly identical, and Year 9 pupils use this to find a missing length or angle [1].

Worked example: Triangle A: opposite 3 cm, hypotenuse 5 cm, ratio = 3 ÷ 5 = 0.6. Similar Triangle B: hypotenuse 10 cm, so opposite = 0.6 × 10 = 6 cm. Scale factor from A to B is 2, so each side doubles.

Ask: The two triangles have angles 90°, 37°, 53°. What will match? (Angles match and corresponding side ratios match because sides stay in the same proportion.) · Which sides must you divide to get the same ratio in both triangles? (The matching sides, for example opposite ÷ hypotenuse in each triangle.)

Check understanding: Thumbs up if ratios 0.6 and 0.6 match; if not, re-point to corresponding sides and repeat division.

Support: Assign roles for measuring talk: Student A reads sides, Student B checks angle match, Student C enters division. **Stretch:** Predict the ratio for a fourth larger triangle with the same angles before calculating.

Use: Slides 5, 7-8, Video clip 2 (0:00–5:00)

Created with LessonDino.com · AI lesson planner with checkable sources

32–52 min **Main activity** 20 min · Pairs

Find unknown sides in similar triangles using scale factor and trig ratios.

YOU DO

1. Hand out 12 triangle cards, squared paper and calculators and show Slide 9 task steps.
2. Direct pairs to sketch each triangle, mark right angle and hypotenuse, then sort into families with same shape.
3. Direct pairs to measure corresponding sides, calculate short side ÷ hypotenuse, and use the common ratio to find a missing side.
4. Circulate to check choice of Pythagoras or matching ratios from the similar twin, then set Worksheet Task D.

STUDENTS DO

1. Sort 12 cards into families, stick matched families together and write the common ratio and multiplier.
2. Sketch each triangle, mark right angle and hypotenuse, and calculate to find missing sides.
3. Complete Worksheet Task D using similar triangles and trig ratios to find sides and angles.

Say: Sort the cards into families that have the same angles. For each family, work out the common ratio and the multiplier. Then choose Pythagoras or matching ratios from the similar twin, calculate the missing side, and check with an estimate.

Worked example: Family with 3-4-5 and 6-8-10: ratio short ÷ hypotenuse = 3 ÷ 5 = 0.6 and 6 ÷ 10 = 0.6. New twin with hypotenuse 15 cm: missing short side = 0.6 × 15 = 9 cm. Check by Pythagoras: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$ [4].

Ask: How do you know two cards belong in the same family? (Same angles, so corresponding sides are in the same proportion and give the same ratio.) · You know the hypotenuse 15 cm and the family ratio 0.6, how do you find the short side? (Multiply 0.6 × 15 = 9 cm, then check with Pythagoras if the other leg is known.)

Check understanding: Check stuck families show same ratio to 2 decimal places; if ratios differ, re-assign matching sides and re-divide.

Support: Highlight the right angle and the same acute angle in the same colour so students can see which sides correspond before dividing. **Stretch:** Students are given only one side and the common ratio for a new triangle in the same family and must work backwards to find the scale factor.

Use: Slide 9, Worksheet Task D

52–60 min **Plenary** 8 min · Whole class

Secure key words and check both skills with a quick quiz.

YOU DO

1. Revisit Slide 10 words: hypotenuse, similar, perpendicular, and take quick definitions.
2. Run Slide 11 vote on most useful problem: ladder, mobile phone screen diagonal, tree height from shadow.
3. Run Slide 12 quick quiz on whiteboards for hypotenuse, short side and right-angled check.
4. Link back to success criteria and set homework.

STUDENTS DO

1. Define key words without using the word itself.
2. Vote and justify which right-triangle problem they would use most.
3. Answer three quiz questions and self-mark.

Say: A rectangle diagonal splits it into two congruent right-angled triangles [4]. Pythagoras can be used to calculate the diagonal size of the television screen if width and height are known [5]. A triangle is right-angled if it satisfies Pythagoras' theorem [4].

Worked example: Sides 9 cm, 12 cm, 15 cm: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$, so yes, right-angled [4].

Ask: Legs 6 cm and 8 cm, hypotenuse? (10 cm because 36 + 64 = 100.) · Hypotenuse 13 cm, one leg 5 cm, other leg? (12 cm because 169 - 25 = 144.) · Are 9 cm, 12 cm, 15 cm right-angled? (Yes, $9^2 + 12^2 = 15^2$ [4].)

Check understanding: Count whiteboard scores 3/3, 2/3; reteach adding versus subtracting squares if short-side question is weakest.

Support: Provide the writing frame: longest side squared = __, sum of other two squares = __, therefore __.
Stretch: Challenge pairs to make a near-miss triple and explain why it fails the check.

Use: Slides 10-12

Common mistakes

Mistake	How to spot it	How to fix it
Adding squares when finding a short side instead of subtracting	Answer larger than hypotenuse	Remind hypotenuse is longest side opposite right angle [3]; do hypotenuse² minus known leg² then square root
Dividing non-matching sides in similar triangles	Ratios in same family do not match	Mark right angle and equal acute angles first, then divide the same positions, using the three trigonometric ratios in similar right-angled triangles [12]
Claiming a triangle is right-angled by look alone	No check calculation shown	Test if $a^2 + b^2 = c^2$; triangle is right-angled only if it satisfies Pythagoras' theorem [4]

Support

- Highlight the right angle and the same acute angle in the same colour so students can see which sides correspond before dividing.
- Give pre-filled tables and writing frames for squaring, adding or subtracting, and square rooting.

Stretch

- Students are given only one side and the common ratio for a new triangle in the same family and must work backwards to find the scale factor.
- Challenge with decimal triples and working backwards from diagonal and width to find height.

For the LSE or teaching assistant

Sit with target pairs in main activity; prompt angle-colour match, check calculator entry for squares and divisions, and scribe ratio to 2 decimal places where needed.

Check for understanding

Question	Answer
Legs 6 cm and 8 cm. Hypotenuse?	10 cm
Hypotenuse 13 cm, one leg 5 cm. Other leg?	12 cm
Sides 9 cm, 12 cm, 15 cm. Right-angled?	Yes, $9^2 + 12^2 = 15^2$

Homework

Finish Worksheet Task D and write one sentence for when to add or subtract squares.

Sources

1. maths yr 9 curriculum · trinity.bexley.sch.uk · https://trinity.bexley.sch.uk/wp-content/uploads/2025/05/maths-yr-9-curriculum.pdf
2. Hounsdown School - Year 9 Curriculum · hounsdown.hants.sch.uk · http://www.hounsdown.hants.sch.uk/Parents/Year-9/
3. Pythagoras' theorem · mathcentre.ac.uk · https://www.mathcentre.ac.uk/resources/uploaded/mc-ty-pythagoras-2009-1.pdf
4. What is Pythagoras' theorem? - BBC Bitesize · bbc.co.uk · https://www.bbc.co.uk/bitesize/articles/zg4jb7h
5. Pythagoras' theorem - Part 1 - KS3 Maths - BBC Bitesize · bbc.co.uk · https://www.bbc.co.uk/bitesize/articles/zf8mp9q
6. Pythagoras' Theorem - CIMT · cimt.org.uk · https://www.cimt.org.uk/sif/trigonometry/t1/interactive/s1.html
7. Pythagoras' theorem in context KS3 | Y9 Maths | Lesson Resources · thenational.academy · https://www.thenational.academy/teachers/programmes/maths-secondary-ks3/units/geometrical-properties-similarity-and-pythagoras-theorem/lessons/pythagoras-theorem-in-context
8. Lesson: Demonstrating Pythagoras' theorem | Oak National Academy · thenational.academy · https://www.thenational.academy/teachers/lessons/demonstrating-pythagoras-theorem
9. Pythagoras | Biography, Philosophy, & Facts | Britannica · britannica.com · https://www.britannica.com/biography/Pythagoras
10. Pythagoras' Theorem - Transum · transum.org · https://www.transum.org/software/SW/Starter_of_the_day/Students/pythagoras.asp
11. pythagoras' theorem | plus.maths.org · plus.maths.org · http://plus.maths.org/content/tags/pythagoras-theorem
12. Pythagoras and trigonometry - KS3 Maths - BBC Bitesize · bbc.co.uk · https://www.bbc.co.uk/bitesize/topics/z93rkqt
13. Pythagorean theorem | Definition & History | Britannica · britannica.com · https://www.britannica.com/science/Pythagorean-theorem
14. BYJU'S Online learning Programs For K3, K10, K12, NEET, JEE ... · byjus.com · https://byjus.com/maths/pythagoras-theorem/
15. Pythagoras Theorem Questions - Math Salamanders · math-salamanders.com · https://www.math-salamanders.com/pythagoras-theorem-questions.html
16. Pythagoras Theorem - Formula, Proof, Examples - Cuemath · cuemath.com · https://www.cuemath.com/geometry/pythagoras-theorem/
17. PYTHAGORAS THEOREM | Dr Austin Maths · draustinmaths.com · https://www.draustinmaths.com/pythagoras
18. Stonehenge Builders Used Pythagoras' Theorem 2,000 Years Before He Was Born · techtimes.com · https://www.techtimes.com/articles/230844/20180621/stonehenge-builders-used-pythagoras-theorem-2-000-years-before-he-was-born.htm
19. Kids Math: Pythagorean Theorem - Ducksters · ducksters.com · https://www.ducksters.com/kidsmath/pythagorean_theorem.php
20. Pythagorean theorem Facts for Kids · kids.kiddle.co · https://kids.kiddle.co/Pythagorean_theorem
21. Stonehenge Builders Used Pythagoras' Theorem 2,000 Years Before He Was Born : ScienceAlert · sciencealert.com · https://sciencealert.com/pythagoras-triangle-used-construction-stonehenge