Fair Coins: Listing Outcomes to Find Probability

Teacher		**Class**	Year 8
Subject	Maths	**Topic**	Probability
Date		**Room**	
Lesson length	60 minutes	**Country**	United Kingdom

Students learn to list theoretical sample spaces for single and combined fair events. They use the lists to calculate probabilities as fractions.

Curriculum links

- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities

Learning objectives

- Students will generate theoretical sample spaces for single and combined fair events.
- Students will calculate theoretical probabilities using favourable divided by total outcomes.

Success criteria

- I can list the sample space for one coin, one die and two coins.
- I can work out theoretical probability as favourable divided by total.
- I can explain why outcomes must be equally likely and separate.

What students already know

Show a coin and a die. Ask: If I toss once, what could happen? Listen for heads, tails and 1 to 6.

Key words

sample space	the full list of everything that could happen in a test [4]
theoretical probability	what you expect, worked out by counting, not by trying it many times
equally likely	each possible result has the same chance of happening [4]
mutually exclusive	only one result can happen at a time, not two together
favourable outcome	a wanted result you count for the event you asked about [4]

Materials

- 1p coins, mini-whiteboards and pens
- Projector, board and Slides 1-6, 9, 11-12
- Video clip 1 Coin and die sample spaces (0:00-4:30) on Slide 6
- Printed Worksheet Task B and Task C, one copy per student
- Squared paper for grids and tree lists

Before the lesson

- Print Worksheet Tasks B and C for each student and spare partially completed HH, HT, TH, TT sheets
- Test Slide 6 Video clip 1 (0:00-4:30) for sound and picture
- Set out 1p coins, mini-whiteboards and pens for pairs and display Slides 1-2

Lesson outline

0–8 min **Starter** 8 min · Whole class

Recall fair single outcomes and hook students into listing two-coin results.

YOU DO

1. Show Slide 1 Predicting the Unpredictable and Slide 2 Heads or six?
2. Hold up two 1p coins and say the hook: I am going to toss two coins - can anyone tell me all the possible results before I toss?
3. Take quick suggestions without confirming, then show Slide 5 What would you test? and take a hands-up vote.
4. Link votes to lesson goal: today we list one-coin and two-coin results and use them to find probabilities.

STUDENTS DO

1. Call out possible results for one coin and one die.
2. Vote for which experiment to try and predict how many results two coins have.
3. Write first guesses for the two-coin list on mini-whiteboards.

Say: A coin toss and a dice roll are fair, so every outcome is equally likely. Today you are detectives. Before I toss, you will list every possible result, then use the list to work out the maths.

Ask: What is the sample space for one fair coin? (Heads, tails - two equally likely results.) · What is the sample space for one fair die? (1, 2, 3, 4, 5, 6 - six equally likely results.) · How many results do you think two coins will have? (Four: HH, HT, TH, TT.)

Check understanding: Hold up mini-whiteboards on one coin and one die. If lists are incomplete, re-point to Slide 2 and correct together before moving on.

Support: Show heads and tails written on the board for students to copy. **Stretch:** Ask: if order matters, why are HT and TH different results?

Use: Slides 1-2, 5

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8–20 min **Teaching** 12 min · Whole class

Model how to list equally likely, separate outcomes and calculate probability.

YOU DO

1. Show Slide 3 Fair means predictable and explain equally likely, mutually exclusive and sample space.
2. Show Slide 4 Single vs combined and model One coin: H, T and Two coins: HH, HT, TH, TT.
3. Play Slide 6 Video clip 1 Coin and die sample spaces (0:00-4:30) and pause to spot the sample space for a coin and for a die.
4. Model P(two heads) = 1 divided by 4 on the board.

STUDENTS DO

1. Copy the H, T and HH, HT, TH, TT lists and the worked probability.
2. Watch the clip and call out the coin and die sample spaces.
3. Chorus the rule: wanted divided by total when results are equally likely.

Say: The sample space is the full list of everything that could happen in one try [4]. For a fair test each result is equally possible, so we work out probability by the number of wanted results divided by the number of all possible results [4]. Probability can be written as a fraction, decimal, or ratio [3].

Worked example: Two fair coins. Step 1: list all equally likely outcomes: HH, HT, TH, TT. Step 2: count total outcomes = 4. Step 3: event one of each = HT, TH, so count wanted = 2. Step 4: calculate 2 divided by 4 = 2/4 = 1/2. So P(one of each) = 1/2.

Ask: Why can we use wanted divided by total here? (Because each outcome is equally possible and only one happens at a time [4].) · What is P(two tails) for two coins? (1/4, because 1 wanted result out of 4 total.)

Check understanding: Thumbs up or down: is HH, HT, TH, TT complete and equally likely? Reteach HT versus TH if unsure.

Support: Keep the HH, HT, TH, TT list on display with first and second coin in different colours. **Stretch:** Ask students to predict the number of outcomes for three coins before telling them 8.

Use: Slides 3-4, 6 and Video clip 1 (0:00–4:30)

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20–40 min **Main activity** 20 min · Pairs

List the one-coin and two-coin sample spaces and calculate probabilities from them.

YOU DO

1. Give out 1p coins and mini-whiteboards and show Slide 9 Your task.
2. Direct pairs to toss one coin 10 times, then stop and write the theoretical sample space: heads, tails.
3. Direct pairs to list all four combined outcomes HH, HT, TH, TT and work out P(two heads), P(one of each) and P(at least one head).
4. Circulate, check lists are systematic and remind students an experiment makes one result you cannot know for sure before it happens [4].

STUDENTS DO

1. Toss one coin 10 times, then write the two-outcome theoretical list.
2. Work in pairs to write the four-outcome two-coin list and three probabilities as fractions.
3. Show the 4-outcome list and probabilities on a mini-whiteboard, for example P(one of each) = 2/4.

Say: Toss one coin 10 times to warm up, then stop tossing and think like mathematicians. List neatly in a table, count the total and the wanted results, and write each probability as a fraction. Probability runs from 0 for impossible to 1 for certain [4].

Worked example: P(at least one head) for two coins. List: HH, HT, TH, TT. Total = 4. Wanted = HH, HT, TH = 3. Probability = 3 divided by 4 = 3/4.

Ask: How do you know your two-coin list is complete? (It has all four: HH, HT, TH, TT, with order kept.) · What is P(two heads) and P(at least one head)? (P(two heads) = 1/4 and P(at least one head) = 3/4.) · What is P(two different results)? (2/4 = 1/2, from HT and TH.)

Check understanding: Scan all mini-whiteboards for the four outcomes and correct fractions. If lists show only three outcomes, pause and fix HT and TH together.

Support: Give a partially completed list with HH and TT filled in, so students only need to find the two mixed outcomes. **Stretch:** Ask students to list the 8 outcomes for three coins and find P(exactly two heads).

Use: Slide 9

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40–52 min **Independent practice** 12 min · On their own

Practise systematic listing for single and combined events on paper.

YOU DO

1. Hand out Worksheet Task B and Task C and recap the chain: list, count total, count wanted, divide.
2. Ask students to complete single events first, listing sample spaces first, then combined outcomes systematically.
3. Circulate and mark two books early to show neat tables and simplified fractions.
4. Stop after 10 minutes and model one combined answer on the board.

STUDENTS DO

1. Complete Worksheet Task B on single events, writing each sample space.
2. Complete Worksheet Task C on combined outcomes, listing systematically then finding probabilities.
3. Check one answer with a partner and simplify fractions.

Say: Work on your own now. List all equally likely outcomes first, then count. Write each answer as a fraction in its simplest form. Show your list so I can see your thinking.

Worked example: Coin + die: outcomes = 2 x 6 = 12. P(heads and 6) = 1 wanted out of 12 total = 1/12. P(heads and any even) = 1 x 3 = 3 wanted out of 12 = 3/12 = 1/4.

Ask: For a coin and a die together, how many outcomes are there? (12 outcomes, because 2 x 6 = 12.) · How do you find the probability of an event when all the outcomes are equally likely? (Count the outcomes that give the event and divide by the total number of outcomes [4].)

Check understanding: Collect books to check one single and one combined list per student. Re-model any missing systematic list on the board if many are incomplete.

Support: Provide a table framework with first picks down the side and second picks across the top to fill in.
Stretch: Ask for P(total is prime) style extra counts from the same grid, simplified to lowest terms.

Use: Worksheet Task B and Worksheet Task C

52–60 min **Plenary** 8 min · Whole class

Secure why 7 beats 2 with two dice and correct faulty sample spaces.

YOU DO

1. Show Slide 11 Think, pair, share: Two dice are rolled. Why is a total of 7 more likely than a total of 2?
2. Take pairs' reasons and stress the 36 equally likely pairs, with six ways to make 7 and one way to make 2 [4].
3. Show Slide 12 True or false? and vote on the three statements about 4 outcomes and coin plus die.
4. Return to success criteria and set homework.

STUDENTS DO

1. Think, pair, share why 7 is more likely than 2 for two dice.
2. Vote true or false and correct the false statement that a coin and a die together have 7 outcomes.
3. Self-assess against I can statements with thumbs up.

Say: We use lists to turn uncertainty into maths. If every pair of dice is equally likely, we count wanted pairs over 36. More ways means higher theoretical probability.

Worked example: Two dice: total outcomes = 36. Ways to make 7 = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) = 6. P(7) = 6/36 = 1/6. Ways to make 2 = (1,1) = 1. P(2) = 1/36. So 7 is more likely.

Ask: Why is a total of 7 more likely than a total of 2? (Because 6 pairs make 7 but only 1 pair makes 2, out of 36 equally likely pairs.) · True or false: a coin and a die together have 7 outcomes? (False, they have 12 outcomes, 2 x 6.)

Check understanding: Vote on Slide 12 statements. If many miss the 12, redraw the 2 by 6 table quickly.

Support: Give the 6 ways to make 7 written out for students to count over 36. **Stretch:** Ask for P(even total) by adding the relevant counts over 36.

Use: Slides 11-12

Common mistakes

Mistake	How to spot it	How to fix it
Listing only HH, HT, TT for two coins, treating HT and TH as the same	Mini-whiteboards show 3 outcomes or P(one of each) = 1/3	Stress order matters for the first and second toss and recount four equally likely outcomes HH, HT, TH, TT [4].
Using wanted divided by total when outcomes are not equally likely	Students say P(red) = 1/4 on an unfair spinner with unequal sections	Show equal versus unequal sections and explain the rule needs each outcome equally possible [4].
Adding instead of multiplying for combined totals, for example coin + die = 7	Slide 12 vote says 7 outcomes or grids show 7 cells	Draw the 2 by 6 table to show 12 cells and count wanted over total [4].

Support

- Give a partially completed list with HH and TT filled in, so students only need to find the two mixed outcomes.
- Provide table and grid frameworks and sentence starters for listing outcomes.

Stretch

- Ask students to list the 8 outcomes for three coins and find P(exactly two heads).
- Challenge students to find P(total more than 9) and P(total is prime) from a two-dice grid.

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For the LSE or teaching assistant

Stay with the main activity pairs. Prompt HT versus TH using two-colour coin cards and check the four-outcome list and simplified fractions.

Check for understanding

Question	Answer
List the sample space for rolling one die.	1, 2, 3, 4, 5, 6
List the sample space for tossing two coins and give P(two heads).	HH, HT, TH, TT; P(two heads) = 1/4
Two dice: what is P(total = 7)?	6/36 = 1/6

Homework

List the 12 outcomes for a coin and a die and find P(heads and 6) and P(heads and an even number).

Sources

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