Estimate First, Prove Exact with Systems

Teacher		**Class**	Grade 8
Subject	Math	**Topic**	Systems of equations
Date		**Room**	
Lesson length	60 minutes	**Country**	United States

Students learn to estimate system solutions by graphing and prove them by algebra. They solve pairs of equations and spot no-solution cases by inspection.

Curriculum links

- Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line [1].
- Algebraic solution methods include elimination and substitution, with systems limited to integer coefficients with at least one coefficient of 1 [2].

Learning objectives

- Students will solve systems of two linear equations by substitution and elimination.
- Students will estimate solutions to systems by graphing both lines.
- Students will decide no solution or infinitely many solutions by inspection.

Success criteria

- I can solve a system by substitution or elimination and check both equations.
- I can graph both lines and estimate the intersection ordered pair.
- I can spot parallel or same-line systems without full solving.

What students already know

Show y = x + 2. Ask: if x = 2, what is y? Then ask: does (2, 4) make x + y = 6 true? 2-minute whiteboard check.

Key words

system	two equations together that use the same x and y at the same time [3]
solution	a pair of numbers that makes every equation true together [3]
intersection	the place where two lines cross on a graph [9]
substitution	putting an expression for one unknown from one equation into the other equation [2]
elimination	adding or subtracting equations to remove one unknown [2]

Materials

- Devices with graphing tool
- Comparison worksheet for graphed estimate next to algebraic answer
- Printed Worksheet Tasks B, C, D, rulers, pencils, graph paper
- Slides 1-12 and whiteboard with markers

Before the lesson

- Print comparison worksheet and Worksheet Tasks B, C, D, one set per pair
- Test Slide 6 Video clip 4 (0:00-4:30) and graphing tool on student devices
- Set axes grids from -6 to 6 ready for integer-solution systems

Lesson outline

0–5 min **Starter** 5 min · Whole class

Activate ideas about finding one pair that fits two equations.

YOU DO

1. Show Slide 1 title Two Lines, One Answer? and read the hook.
2. Show Slide 2 vote with three choices for how to find what fits both.
3. Take a quick vote and ask two voters to explain their choice.
4. Preview that graphing gives a fast guess and algebra gives proof.

STUDENTS DO

1. Listen to the hook and read the vote choices.
2. Vote for graph, plug in, or add or subtract.
3. Explain choice to a neighbor.

Say: Graph it fast for a guess, then solve it clean for proof. Which answer do you trust more? Today we will do both. Two equations share x and y, and we need the pair that fits both.

Ask: If a pair works in the first equation but fails in the second, did we solve the system? (No, a system solution must make every equation true at the same time.) · What could two lines do on a graph? (Cross once, stay parallel, or lie on top of each other.)

Check understanding: Count votes and listen for both-equations idea; if missing, restate that solution must satisfy both before moving on.

Support: Offer word bank: both, cross, plug in, add. **Stretch:** Ask which method would be fastest if you only had 3x + 2y = 5 and 3x + 2y = 6.

Use: Slides 1–2

5–15 min **Teaching** 10 min · Whole class

Explain what a system solution means and the three line pictures.

YOU DO

1. Show Slide 3 systems map and name the three cases for pairs of lines.
2. Show Slide 4 fact that 3x + 2y cannot be 5 and 6 at the same time.
3. Model testing (4, 7) in 2x + y = 15 and 3x - y = 5 on the board.
4. Use Slide 5 question to stress checking the second equation.

STUDENTS DO

1. Copy the three line pictures: one crossing, parallel, same line.
2. Check each substitution step with the teacher.
3. Answer Slide 5 with a partner: what must you check next?

Say: A system is two equations that must be true together with the same x and y. Systems relate to pairs of lines that intersect, are parallel, or are the same line [1]. A solution must make all the equations true at the same time [3].

Worked example: Test (4, 7) in 2x + y = 15 and 3x - y = 5: Step 1: 2(4) + 7 = 8 + 7 = 15, true. Step 2: 3(4) - 7 = 12 - 7 = 5, true. Step 3: both true, so (4, 7) is the solution, represented by an ordered pair (x, y).

Ask: If (4, 7) fits 2x + y = 15, are we done? (No, we must also check 3x - y = 5 because both must be true.) · Why does 3x + 2y = 5 and 3x + 2y = 6 have no solution? (The same left side cannot equal 5 and 6 at once, so graphs are parallel with no crossing.)

Check understanding: Mini whiteboards: yes or no, is (3, 2) a solution to 2x + y = 8 and x - y = 1? Reteach checking both if many say no.

Support: Give sentence frame: It works in the first equation because ___ but fails in the second because ___. **Stretch:** Ask why one equation alone has many pairs but two equations can narrow it to one.

Use: Slides 3–5

15–27 min **Graphing estimate** 12 min · Whole class

Teach estimating the solution as the intersection point.

YOU DO

1. Show Slide 6 and play Video clip 4 (0:00–4:30) on solving by graphing.
2. Model graphing y = x + 2 and x + y = 6 on the same grid.
3. Circle the crossing at (2, 4) and check it in both equations.
4. Hand out Worksheet Task D and set estimate-only goal.

STUDENTS DO

1. Watch the clip for how intersection gives x and y.
2. Copy the two graphed lines and circle (2, 4).
3. Start Worksheet Task D: complete tables, draw both lines, estimate the crossing.

Say: We can find the solution by graphing the equations and finding the point of intersection [9]. The graphs of linear equations are lines, and some pairs have one crossing, some have none, and some have many [6]. Remember a non-integer crossing gives only an estimate.

Worked example: Graph y = x + 2 and x + y = 6: make tables, plot both lines, they cross at (2, 4). Check: 4 = 2 + 2 true, 2 + 4 = 6 true, so estimate is exact here.

Ask: What does the crossing point tell us? (The x and y that make both equations true.) · When is graphing only an estimate? (When lines cross between grid lines or the picture is not exact.)

Check understanding: Thumbs up on estimated pair for Task D first graph; regraph one line together if estimates scatter.

Support: Provide pre-labeled grids from -6 to 6 with one line already graphed. **Stretch:** Ask students to name a system whose crossing is not on a grid point and explain the limit.

Use: Slide 6, Video clip 4 (0:00–4:30), Worksheet Task D

27–50 min **Main activity** 23 min · Pairs

Compare graphed estimates with exact substitution, elimination, and inspection.

YOU DO

1. Show Slide 7 to contrast graphing estimate with exact algebra.
2. Model System A by substitution and System B by elimination, then System C by inspection using Slide 9.
3. Hand out comparison worksheet and Worksheet Tasks B and C for pairs to solve.
4. Circulate, prompt checking both equations, and collect estimate versus exact pairs.

STUDENTS DO

1. Graph quickly or use device to record an estimate for A and B.
2. Solve A by substitution in Task B and B by elimination in Task C, then decide C by inspection.
3. Write estimate, exact pair, and both-equation checks on the comparison sheet.

Say: Graphing shows lines cross for a fast estimate of the solution. Substitution or elimination gives the exact pair that fits both [2]. You will record the graphed guess next to the algebra answer and confirm the exact pair makes both equations true.

Worked example: A y = x + 2, x + y = 6 by substitution: x + (x + 2) = 6, 2x + 2 = 6, 2x = 4, x = 2, y = 4. B 2x + y = 15, 3x - y = 5 by elimination: add to get 5x = 20, x = 4, then 2(4) + y = 15, y = 7. C 3x + 2y = 5, 3x + 2y = 6 by inspection: same left side, different totals, so no solution.

Ask: After you find x = 4, are you done? (No, substitute back to find y and check both original equations.) · How do you know C has no solution without graphing? (3x + 2y cannot be 5 and 6 at the same time, so lines are parallel.) · Which answer do you trust more, graph or algebra? (Algebra, because graphing can be off and algebra proves the exact pair.)

Check understanding: Collect one pair's A, B, C answers on the board and check both equations; reteach the missed move in small group if needed.

Support: Provide integer-solution systems with graphs that land exactly on grid points. **Stretch:** Try a system with parallel lines, show empty intersection on the graph, and prove no solution by inspection.

Use: Slides 7, 9, Worksheet Tasks B, C

50–60 min **Plenary** 10 min · Whole class

Confirm exact solutions, inspection cases, and the big idea.

YOU DO

1. Show Slide 10 statement that a system solution must make every equation true at the same time.
2. Show Slide 11 mystery about paths that never meet and name no solution.
3. Run Slide 12 true or false on (4, 7) and on 3x + 2y = 5 and 6.
4. Give exit ticket on paper.

STUDENTS DO

1. Vote true or false with reasons for Slide 12 statements.
2. Call out one solution, no solution, or many for each example.
3. Complete three-question exit ticket on their own.

Say: A consistent system has at least one solution and an inconsistent system has no solution [4]. Parallel lines never share a point, so that system has no solution. Tell me in five seconds: same left side, different total, could both be true?

Ask: Two straight paths run side by side forever and never meet. What about their system? (No solution because parallel lines never share a point.) · Why must we check both equations? (Solutions to a system are values that make both equations true.)

Check understanding: Exit tickets sorted into correct and not yet; pull small group tomorrow if checking second equation is missing.

Support: Let students point to graphed crossing while stating true or false. **Stretch:** Ask students to write their own no-solution pair with integer coefficients.

Use: Slides 10–12

Common mistakes

Mistake	How to spot it	How to fix it
Thinking a pair that fits one equation already solves the system	Student checks only first equation, as in (4, 7) fitting 2x + y = 15 but not tested in 3x - y = 5	Require two-line check table; a solution must make all equations true at the same time [3].
Treating a graphed estimate as exact when lines cross between grid lines	Estimate like (2.5, 3.5) read as (2, 3) or (3, 4) without algebra	Solve by substitution or elimination after graphing to prove the exact pair [2].
Sign error when adding or subtracting to eliminate a variable	With 2x + y = 15 and 3x - y = 5, subtracting instead of adding or losing the minus	Line up y-terms, circle opposites, and check by putting the pair into both original equations.

Support

- Provide integer-solution systems with graphs that land exactly on grid points.
- Give substitution template with blanks for left side, right side, and true or false.
- Provide equations already written with one variable solved, like y = 2x - 1.

Stretch

- Students try a system with parallel lines, show empty intersection on the graph, and prove no solution by inspection.
- Students write their own no-solution and infinitely many solutions pairs using integer coefficients.
- Students explain why graphing gives only an estimate for a non-integer intersection.

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For the LSE or teaching assistant

Stay with target pairs in main activity; prompt second-equation check, help line up tables for graphing, and read Task B and C steps aloud.

Check for understanding

Question	Answer
Solve by substitution: y = x + 2 and x + y = 6. What is the solution?	(2, 4)
Solve by elimination: 2x + y = 15 and 3x - y = 5. What is the solution?	(4, 7)
By inspection, how many solutions does 3x + 2y = 5 and 3x + 2y = 6 have? Why?	No solution because 3x + 2y cannot be both 5 and 6.

Homework

Solve two given systems, one by substitution and one by elimination, and check both equations.

Sources

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