Find Missing Sides in 2D and Inside Boxes

Teacher		**Class**	Grade 8
Subject	Math	**Topic**	The Pythagorean theorem
Date		**Room**	
Lesson length	60 minutes	**Country**	United States

Students learn to use $a^2 + b^2 = c^2$ for legs, hypotenuses, and box diagonals. They solve ladder, screen, and delivery-box challenges in pairs.

Curriculum links

- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions [2]
- Analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understand and apply the Pythagorean Theorem [1]

Learning objectives

- Students will find missing hypotenuse and leg lengths with $a^2 + b^2 = c^2$.
- Students will solve real-world 2D problems with ladders, screens, and shortcuts.
- Students will use two steps to find space diagonals inside rectangular boxes.

Success criteria

- I can spot the hypotenuse and choose to add or subtract.
- I can solve for any missing side and check my answer.
- I can find base and space diagonals in 3D boxes.

What students already know

Ask: what is 6^2, 8^2, 10^2, and $\sqrt{64}$, $\sqrt{100}$, $\sqrt{25}$? Show 3 triangles; students point to the right angle and longest side.

Key words

hypotenuse	the longest side in a right triangle, across from the right angle [10]
leg	one of the two shorter sides that meet to make the right angle [4]
right triangle	a triangle that has one corner exactly 90 degrees [10]
space diagonal	a straight line through the inside of a box from bottom corner to opposite top corner
square root	the number that times itself makes the given number

Materials

- rulers
- calculators
- box nets or empty small boxes
- colored pencils
- Slides 1, 2, 3, 4, 5, 7, 9, 10, 11, 12
- Worksheet Task B and Task C printed for each pair

Before the lesson

- Print Worksheet Task B and Task C for each pair and a two-box template for support
- Collect small boxes, rulers, calculators, and colored pencils for measuring in inches
- Load Slides 1, 2, 3, 4, 5, 7, 9, 10, 11, 12 and check projector

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- two-box template labeled Step 1 base and Step 2 height to space diagonal
- Pre-measure one example box 6 in. by 8 in. by 5 in. to check answers

Lesson outline

0–8 min **Starter** 8 min · Whole class

Hook students with the delivery-box problem and recall squares and right triangles.

YOU DO

1. Show Slide 1 title and Slide 2 objectives, then tell the hook story about mailing the tube.
2. Show Slides 4 cards and ask where right triangles hide in ladders and boxes.
3. Quick-fire squares and roots: 3^2, 4^2, 5^2, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$.
4. Take predictions: could a 15-inch tube fit in a 12-inch box diagonally?

STUDENTS DO

1. Listen, look at the box, and give thumbs up or down for fit.
2. Call out squares and roots and point to right angles on Slides 4.
3. Share one sentence with a partner about why a diagonal is longer than a side.

Say: You need to mail a 15-inch art tube and you only have a 12-inch box. Could it fit diagonally through the inside? Today we will learn a rule that finds hidden straight lines without measuring them. We will test your idea by measuring and calculating.

Ask: Why might the inside diagonal be longer than any edge of the box? (It slants across, so it stretches from corner to far corner.) · What is $3^2 + 4^2$, and what square root undoes it? ($9 + 16 = 25$, and $\sqrt{25} = 5$.)

Check understanding: Use thumbs up, sideways, down for: I know squares and roots. Reteach $6^2=36$ and $\sqrt{36}=6$ if many are unsure.

Support: Show a 3-4-5 triangle with squares 9, 16, 25 and let students count to see $9 + 16 = 25$. **Stretch:** Ask what $5^2 + 12^2$ predicts for the hypotenuse before calculating.

Use: Slides 1, 2, 4

8–20 min **Teaching** 12 min · Whole class

Teach students to label c correctly and decide when to add or subtract.

YOU DO

1. Show Slides 3 compare and point to c on 4 rotated triangles.
2. Show Slides 5 mystery with 9 + 16 = 25 and link to $a^2 + b^2 = c^2$, with c being the hypotenuse [3].
3. Model Slides 7 steps with the 5-foot out, 13-foot ladder example.
4. Emphasize check: c must be longest.

STUDENTS DO

1. Point to c on each diagram and say why the other sides cannot be c.
2. Copy the 5-13 steps and chant add-for-c, subtract-for-leg.
3. Show on whiteboards whether each new problem wants c or a leg.

Say: For any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides [8]. The hypotenuse is the longest side, opposite the right angle [10]. If you want c, you add. If you know c and want a leg, you subtract from c^2, then square root.

Worked example: 13-foot ladder, foot 5 feet out: mark right angle, label ladder c=13, leg a=5. Write $a^2 + b^2 = c^2$ [3]. Plug in: $5^2 + b^2 = 13^2$. So $25 + b^2 = 169$. Subtract: $b^2 = 169 - 25 = 144$. Square root: b = 12 feet. Check: 12 is less than 13, so the leg is shorter than the hypotenuse [10].

Ask: How do you know which side is c before you calculate? (It is opposite the right angle and must be the longest side [10].) · When do you subtract instead of add? (When the unknown is a leg, do c^2 - known leg^2, then square root.)

Check understanding: Mini-whiteboards: hold up c circled on two new triangles. Re-model with right-angle card if many miss.

Support: Give a small right-angle card to place on the diagram; the side that does not touch the corner is c [10]. **Stretch:** Ask what goes wrong with 6-8-10 if c is mixed up: show $6^2 + 10^2$ does not equal 8^2.

Use: Slides 3, 5, 7

20–32 min **Guided Practice** 12 min · Whole class

Practice finding a leg and a hypotenuse in 2D word problems together.

YOU DO

1. Show Slides 10 photo of 10-foot ladder 6 feet from wall and draw the right triangle.
2. Solve together using Worksheet Task B routine: circle hypotenuse, add or subtract.
3. Repeat for 6 in. by 8 in. screen diagonal where unknown is c.
4. Cold-call for each step and units.

STUDENTS DO

1. Complete Worksheet Task B rows with teacher, circling c each time.
2. Write equation, subtraction or addition step, square root, and units.
3. Compare answers with a partner and check c is longest.

Say: We circle the hypotenuse first, then add or subtract. In a right triangle the squares of the two short sides add to the square of the long side [4]. We always finish with a square root and write feet or inches.

Worked example: Ladder leg: $6^2 + b^2 = 10^2$, so $36 + b^2 = 100$, $b^2 = 64$, $b = 8$ feet. Screen hypotenuse: $6^2 + 8^2 = c^2$, so $36 + 64 = 100$, $c^2 = 100$, $c = 10$ inches. Both check because the hypotenuse 10 is longest [10].

Ask: For the ladder, why do we do 100 - 36 and not 100 + 36? (The unknown is a leg, so leg² = c² - other leg².) · For the screen, why do we add 36 + 64? (The unknown is the hypotenuse, so $c^2 = 36 + 64$ [3].)

Check understanding: Check Task B lines for circled c and correct 8 feet and 10 inches. Pull small group to re-label if c is wrong.

Support: Give equation starter ____² + 6² = 10² and a multiplication chart for squares. **Stretch:** Change numbers to give non-whole root and round to nearest tenth inch.

Use: Slides 10, Worksheet Task B

32–52 min **Main Activity** 20 min · Pairs

Apply the theorem in two steps to find the longest rod that fits in a box.

YOU DO

1. Show Slides 9 task and Slides 11 question about 6 in. by 8 in. box 5 in. tall.
2. Hand out boxes, rulers, calculators, colored pencils, and Worksheet Task C.
3. Model Step 1 base diagonal then Step 2 space diagonal with color coding.
4. Circulate, prompt pairs to list both equations with units to nearest tenth inch.

STUDENTS DO

1. Measure length, width, height in inches and draw base and space diagonals in color.
2. Solve Slides 9 three challenges and Worksheet Task C, rounding to nearest tenth.
3. Write both equations, circle c each step, and report longest rod with units.

Say: First find the base diagonal with $a^2 + b^2 = c^2$, with c being the hypotenuse [3]. Then use that diagonal as one leg and the box height as the other leg for a second right triangle up through the box. This two-step use applies the theorem to determine unknown side lengths in three dimensions [2]. Draw the base diagonal and the space diagonal in color.

Worked example: Box 6 in. by 8 in., height 5 in. Step 1 base: $6^2 + 8^2 = d^2$, $36 + 64 = 100$, $d = 10$ in. Step 2 space: $10^2 + 5^2 = s^2$, $100 + 25 = 125$, $s = \sqrt{125} \approx 11.2$ in. So an 11-inch pencil fits, a 12-inch pencil does not.

Ask: What length becomes a leg in Step 2? (The base diagonal from Step 1 becomes a leg with height as the other leg.) · How do you know your space diagonal answer is reasonable? (It must be longer than base diagonal and all edges, since hypotenuse is longest [10].)

Check understanding: Collect sticky-note answers for base 10 in. and space 11.2 in.; re-teach Step 1 to Step 2 link if many stop at 10.

Support: Provide a two-box template labeled Step 1 base and Step 2 height to space diagonal, with the hypotenuse as the longest side opposite the right angle [10]. **Stretch:** Ask for the largest whole-inch rod that fits and whether a slightly taller box with the same base would help more than a wider base.

Use: Slides 9, 11, Worksheet Task C

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52–60 min **Plenary** 8 min · Whole class

Check understanding of hypotenuse, legs, and 2D and 3D uses and fix errors.

YOU DO

1. Show Slides 12 true or false and vote on each statement.
2. Fix one error card: $5^2 + 12^2 = 169$ feet missing root.
3. Return to hook: explain why 15-inch tube can fit in 12-inch box diagonally.
4. Set homework and collect Task C.

STUDENTS DO

1. Vote true or false and defend in one sentence.
2. Correct the error by crossing out 169 feet and writing $\sqrt{169} = 13$.
3. Write exit ticket answers on slips.

Say: In $a^2 + b^2 = c^2$, c is always the hypotenuse [3]. It only works for right triangles. The area of the square on the hypotenuse equals the area of the square on side a plus the area on side b [8]. If you get 169, you are not done until you square root to 13.

Ask: True or false: c is opposite the right angle and longest? (True [10].) · A student writes $5^2 + 12^2 = 169$, so answer 169. What is missing? (Square root and units: $\sqrt{169} = 13$.)

Check understanding: Exit slips for 3 questions; if many miss leg vs hypotenuse, start next lesson with re-label drill.

Support: Provide finished example where 169 becomes 13 after square root with c marked longest [10].
Stretch: Add leg-unknown error where students must show $c^2 - b^2 = a^2$ then root and check a is less than c.

Use: Slides 12

Common mistakes

Mistake	How to spot it	How to fix it
Forgetting to take the square root and giving c^2 as length	Answer is 64, 100, or 169 instead of 8, 10, or 13, often missing units	Require last line $\sqrt{__} = __$ with units; show $9 + 16 = 25$ needs $\sqrt{25} = 5$ to get side from area [8].
Labeling a leg as c, so adding when should subtract	Leg answer is longer than given hypotenuse, like leg = 11 when c = 10	Circle hypotenuse first; hypotenuse is longest side opposite right angle [10], and use $a^2 + b^2 = c^2$ with c as hypotenuse [3].
Using the theorem on a non-right triangle	Applies $a^2 + b^2 = c^2$ to any triangle, e.g., 7-8-9 as if right	Check for right-angle mark first; test $7^2 + 8^2 = 113$, not 81, so not right since squares of short sides must add to square of long side [4].

Support

- Two-box template for Step 1 base and Step 2 height to space diagonal
- Multiplication chart and equation starters like $____^2 + 5^2 = 13^2$
- Right-angle card to find c as side not touching the corner

Stretch

- Largest whole-inch rod that fits and taller vs wider box comparison
- Non-whole answers rounded to nearest tenth and simplest radical form
- Make own triple by doubling 3-4-5 and test with $a^2 + b^2 = c^2$

For the LSE or teaching assistant

Sit with pairs using template; prompt circle c, read squares from chart, check calculator square-root step, and confirm units in inches or feet.

Check for understanding

Question	Answer
A 10-foot ladder is 6 feet from the wall. How high up the wall?	8 feet ($6^2 + b^2 = 10^2$, b = 8)
Screen 6 in. by 8 in. What is the diagonal?	10 inches (36 + 64 = 100, $\sqrt{100} = 10$)
Box 6 in. by 8 in., height 5 in. Longest pencil inside to nearest tenth?	11.2 inches (base 10 in., $\sqrt{(100+25)}=\sqrt{125}\approx 11.2$)

Homework

Find to the nearest tenth inch the longest rod in a 9 in. by 12 in. box 8 in. tall.

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