Three Rungs to X: Multi-Step Angle Equations

Teacher		**Class**	Grade 7
Subject	Math	**Topic**	Angles
Date		**Room**	
Lesson length	60 minutes	**Country**	United States

Students learn to link complementary, supplementary, vertical and adjacent facts in a chain. They climb a three-rung puzzle to write equations and solve for x.

Curriculum links

- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. [2]

Learning objectives

- Students will identify complementary, supplementary, vertical and adjacent angle pairs.
- Students will write simple equations for an unknown angle in a figure.
- Students will solve multi-step angle problems and check their answers.

Success criteria

- I can name the angle pair and pick 90, 180, or equal.
- I can write an equation for the unknown angle.
- I can solve the chain and check each step.

What students already know

Show a square corner and a straight line. Ask: How many degrees in the corner? How many on the line? Listen for 90 and 180.

Key words

supplementary	Two angles that together make a straight line, totaling 180 degrees [6].
complementary	Two angles that together make a square corner, totaling 90 degrees [8].
vertical	Angles opposite each other where two lines cross; they have the same size [19].
adjacent	Angles next to each other that share a side and a corner point [6].

Materials

- Slides 1-7, 9-12
- Video clip 1 (1:50–6:49) on slide 6
- Printed ladder sheets for Ladder to X
- Pencils
- Mini whiteboards and markers
- Worksheet Task B copies

Before the lesson

- Print ladder sheets, one per student, with Rung 1 L-shape, Rung 2 X, Rung 3 straight line
- Print Worksheet Task B, one per pair
- Load slides 1-7, 9-12 and test Video clip 1 (1:50–6:49) with sound
- Set out pencils, whiteboards and markers

Lesson outline

0–7 min **Starter** 7 min · Whole class

Hook students and recall 90-degree corners and 180-degree lines for equation work.

YOU DO

1. Show Slide 1 title and Slide 2 objectives and read them in student words.
2. Deliver the hook with energy and hold up a blank ladder sheet.
3. Show Slide 5 question: Two lines cross. One angle is 72 degrees.
4. Ask pairs to discuss how to find the other three without a protractor.

STUDENTS DO

1. Listen to objectives and hook.
2. Think, then pair and share ideas about the 72-degree cross.
3. Share one idea with the class.

Say: No phones, no tablets. Just paper, pencil, and three rungs to climb. Look at the X. If one opening is 72 degrees, talk with your partner: how could you figure out the other three openings without measuring them all?

Ask: If two lines cross, what do you know about the angle opposite 72 degrees? (It is also 72 degrees because opposite angles when lines cross are equal [19].) · What about the angle right next to 72 degrees? (It must add with 72 to make 180 because they sit on a straight line [6].)

Check understanding: Quick thumbs up: can you find at least one other angle? If many are unsure, point to the straight line and the X shape before moving on.

Support: Show a sketched X with 72 labeled and point to opposite and next-door positions. **Stretch:** Ask what equations they would write for the two unknown neighbors.

Use: Slides 1-2, 5

7–19 min **Teaching** 12 min · Whole class

Teach the four angle facts and how each fact becomes an equation.

YOU DO

1. Show Slides 3-4 and have students repeat each pair and total.
2. Play Video clip 1 on Slide 6 and pause to label opposite equal and next-door 180.
3. Show Slide 7 Solver Steps and model Name it, Link it, Solve it.
4. Model the scissors X with 38 degrees and solve for both unknowns on the board.

STUDENTS DO

1. Chorally name totals: 90 for corner, 180 for line, equal for X.
2. Watch video and call out opposite and next-door angles.
3. Copy the 38-degree X model with both equations.

Say: Name it first. Is it a square corner, a straight line, or an X? Then link it: corner means equals 90, line means equals 180, X opposite means equal. A straight angle measures 180 degrees [6], and complementary angles add to 90 degrees [8].

Worked example: Two lines cross, one angle is 38 degrees. Name it: next-door angles share a side [6] and form a linear pair that adds to 180 degrees [6]. Link it: x + 38 = 180. Solve it: subtract 38 from both sides, x = 142. The angle across from 38 is vertical, and opposite angles are equal [19], so it is 38 degrees. The angle across from x is 142 degrees.

Ask: Why do we write x + 38 = 180 for next-door angles on a cross? (Because next-door angles on a cross make a straight line and supplementary angles sum to 180 degrees [6].) · How do we know the angle across from 38 without another equation? (They are vertical and opposite angles are equal [19], so it is 38 degrees.)

Check understanding: Whiteboards up: write the equation for a 38-degree X. Reteach link to 180 if students write 90.

Support: Give fill-in frame x + 38 = 180 and let students solve only for x first. **Stretch:** Ask what changes if the 38 is labeled 2x + 8 and solve the two-step.

Use: Slides 3-4, 6-7; Video clip 1 (1:50–6:49)

19–30 min **Guided Practice** 11 min · Whole class

Practice writing and solving one- and two-part equations together.

YOU DO

1. Show Slide 9 first example: straight line shows 38, x plus 22, and 3x.
2. Think aloud to combine terms and solve, then check by adding back to 180.
3. Show Slide 9 second example: lines cross, one angle 72, opposite is x plus 18.
4. Have class call out each step for the vertical equation.

STUDENTS DO

1. Solve each step on whiteboards with the teacher.
2. Start Worksheet Task B first two problems with a partner.
3. Compare equations, not just answers.

Say: Three angles sit side by side on a straight line, and adjacent angles share a side [6]. That means all three must total 180. For the cross, opposite means equal, so set the expressions equal and solve.

Worked example: First example: 38 + (x + 22) + 3x = 180 because angles on a straight line add to 180 degrees [6]. Combine: 4x + 60 = 180. Subtract 60: 4x = 120. Divide by 4: x = 30. Check: 38 + 52 + 90 = 180. Second example: x + 18 = 72 because opposite angles are equal [19]. Subtract 18: x = 54.

Ask: Why is the straight-line example 38 + x + 22 + 3x = 180 and not = 90? (They sit on a straight line, and supplementary angles sum to 180 degrees [6].) · Why is the crossed-lines example x + 18 = 72? (The two angles are opposite when lines cross, and opposite angles are equal [19].)

Check understanding: Scan whiteboards for correct totals, 180 vs 90 vs equal. Pull back to Slide 4 90 or 180 comparison if mixed up.

Support: Highlight the straight-line mark and the X, and provide equation starters ___ = 180 and ___ = ___.
Stretch: Ask students to find the actual degree values of x + 22 and 3x after x = 30.

Use: Slides 7, 9; Worksheet Task B

30–50 min **Main Activity** 20 min · Pairs

Solve a three-step chain that uses complement, vertical, and supplement in order.

YOU DO

1. Hand out printed ladder sheets and pencils and show the three rungs.
2. Direct Rung 1: right-angle corner with 27 degrees and a, write a + 27 = 90.
3. Direct Rung 2: use a as a vertical match to get b, then Rung 3 b next to x on a line.
4. Circulate, check equations, and push finishers to the stretch.

STUDENTS DO

1. Work in pairs to write all three equations and solve for a, b, then x.
2. Box answers and check by adding back to 90 and 180.
3. Swap with another pair to verify the chain.

Say: Climb in order. You cannot skip a rung. Rung 1 is a square corner, so complementary angles add to 90 degrees [8]. Rung 2 is an X, so opposite angles are equal [19]. Rung 3 is a straight line, so supplementary angles sum to 180 degrees [6]. Box a, b, and x and show all three equations.

Worked example: Rung 1: a + 27 = 90, so a = 63. Rung 2: b is opposite a, so b = 63 because opposite angles are equal [19]. Rung 3: b and x are adjacent and share a side [6] on a straight line, so b + x = 180. Substitute: 63 + x = 180, so x = 117. This completes a multi-step problem to write and solve simple equations for an unknown angle in a figure [2].

Ask: After you find a = 63, how do you get b with no new math? (b is vertical to a, and opposite angles are equal [19], so b = 63.) · What equation finds x from b? (b + x = 180 because they are supplementary on a straight line [6].)

Check understanding: Collect ladders and check for a = 63, b = 63, x = 117 with three equations. Re-model any rung where the total is wrong.

Support: Pre-fill the first equation a + 27 = 90 and let students solve and carry forward. **Stretch:** Change Rung 3 to 2x + b on the line for a two-step finish: 2x + 63 = 180.

50–60 min **Plenary** 10 min · Whole class

Review best first moves and prove learning against success criteria.

YOU DO

1. Show Slide 10 fact: full circle is 360, half turn is 180.
2. Run Slide 11 vote: Best First Move when stuck on a puzzle.
3. Show Slide 12 exit ticket and have students write equation and answer.
4. Close by linking votes to success criteria.

STUDENTS DO

1. Vote for first move and defend with 90, 180, or equal.
2. Complete exit ticket with equation and check.
3. State one thing learned and one question still open.

Say: A full circle is 360 equal degrees [3]. Half a turn is 180 degrees, so angles on a straight line add to 180. Good solvers look first for the square corner, the straight line, or the X, then write the matching equation.

Ask: When should your first move be to look for the X? (When you see two crossing lines, because opposite angles are equal [19].) · When should your first move be the straight line? (When angles sit side by side on a line, because they sum to 180 degrees [6].)

Check understanding: Read exit tickets for correct equation choice and solution. Reteach 90 vs 180 tomorrow if many mix them.

Support: Allow pointing to Slide 4 corner vs line pictures to choose the vote. **Stretch:** Ask students to write a one-sentence rule for when adjacent angles give an equation.

Use: Slides 10-12

Common mistakes

Mistake	How to spot it	How to fix it
Using 90 instead of 180 for angles on a straight line, or the reverse.	Equation shows = 90 for a straight-line trio, or = 180 for a square corner.	Mark the square corner and straight line. Ask: Is this a corner family that adds to 90 degrees [8] or a line family that adds to 180 degrees [6]?
Treating adjacent angles as always equal like vertical angles.	Student sets next-door angles equal instead of adding them.	Show the X. Adjacent angles share a side [6] and add to 180, while only opposite angles are equal [19]. Rewrite as sum = 180.

Support

- Pre-fill the first ladder equation a + 27 = 90 and let students solve and carry forward.
- Provide equation starters and color key for 90-degree corner and 180-degree line.

Stretch

- Change Rung 3 to 2x + b on the line for a two-step finish.
- Require combining terms like x + 2x or solving 3x + 2x on a line.

For the LSE or teaching assistant

Sit near pairs who mix 90 and 180. Prompt Name it then Link it, point to corner and line marks, and check that a and b are carried correctly to Rung 3.

Check for understanding

Question	Answer

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A square corner has 38 degrees and x. Write the equation and find x.	x + 38 = 90, x = 52.
Two lines cross. One angle is 72 degrees, the opposite is x + 18. Find x.	x + 18 = 72, x = 54.
Angles 38, x + 22, and 3x sit on a straight line. Write the equation and find x.	38 + x + 22 + 3x = 180, x = 30.

Homework

Finish Worksheet Task B and check each answer by adding back to 90 or 180.

Sources

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