Pack It, Multiply It: Volume With Fractions

Teacher		**Class**	Grade 6
Subject	Math	**Topic**	Volume of rectangular prisms
Date		**Room**	
Lesson length	60 minutes	**Country**	United States

Students learn that packing with fractional unit cubes gives the same volume as multiplying edge lengths. They pack, multiply, and solve real box problems with fractions.

Curriculum links

- Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. [1]
- Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. [1]

Learning objectives

- Students will pack prisms with fractional unit cubes to find volume.
- Students will explain why packing matches multiplying edge lengths.
- Students will use V = l w h and V = B h with fractions.

Success criteria

- I can pack a prism with fractional cubes and count the volume.
- I can show packing gives the same answer as multiplying edges.
- I can use V = l w h and V = B h for fractional prisms.

What students already know

Hold up a 3 by 2 by 2 box built from unit cubes: How many cubes fill one layer? How many layers? What is the total volume?

Key words

volume	amount of space taken up inside a solid shape, counted in cubes [3][10]
rectangular prism	box shape with six flat rectangle sides and square corners
unit cube	small block for filling space, with tiny edge like 1/2 inch [1]
edge length	how long one straight side of a box or cube is
base area	size of the bottom face, found by length times width [17]
cubic units	small cubes used to state how much space is filled [10]

Materials

- 1/2-inch cubes
- small empty boxes
- ruler
- recording sheet

Before the lesson

- Print recording sheet and Worksheet Task B and Task C, one per pair
- Test Slide 5 Video clip 2 (0:00–5:00) for sound and picture

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- Slides 1-6 and 8-11
- Video clip 2 (0:00–5:00)
- Worksheet Task B and Task C
- whiteboards and markers
- Set out trays with 1/2-inch cubes, small empty boxes, rulers
- Tape one sample 2 1/2 by 2 in rectangle on a tray for demo

Lesson outline

0–5 min **Starter** 5 min · Whole class

Recall whole-number volume and create a need for fractional cubes.

YOU DO

1. Show Slides 1-3 and a small empty box that is 2 and 1/2 inches long.
2. Ask the hook and try one whole unit cube inside to show the gap.
3. Elicit that volume is how many cubic units fit inside and that whole cubes will not fill it [3][10].
4. Preview the big idea that tiny fraction cubes give the same answer as multiplying.

STUDENTS DO

1. Look at the box and Slides 1-3.
2. Discuss with a partner what size tiny cubes would fill it with no gaps.
3. Share ideas that halves could fit exactly.

Say: This box measures 2 and 1/2 inches long. Whole cubes leave gaps — what size cubes would fill it exactly? Turn and tell your partner. Today we will pack boxes with tiny cubes, then prove multiplying gives the same answer.

Ask: What does volume tell us about a box? (How many cubic units fit inside the solid [3], measured in cubic units [10].) · Why will whole-inch cubes not fill a 2 and 1/2 inch edge exactly? (The 1/2 left over leaves a gap; a 1/2-inch cube would fill it.)

Check understanding: Thumbs up if you can say what volume counts. If unsure, rebuild a 3 by 2 whole-number layer together before moving on.

Support: Show a whole-number box built out of unit cubes, such as by building the prism out of cubes [20], then point to the leftover 1/2 gap. **Stretch:** Ask how many 1/2-inch cubes fit along 2 and 1/2 inches before teaching.

Use: Slides 1-3

5–17 min **Teach - Packing to Multiplying** 12 min · Whole class

Model how counting fractional cubes matches multiplying edge lengths.

YOU DO

1. Show Slide 4 tools, then play Video clip 2 on Slide 5 to model counting 1/2-inch cubes on each edge.
2. Talk through Slide 6 chain: 5 cubes by 4 cubes in a layer, 3 layers, then convert to cubic inches.
3. Model multiplying the three edge lengths to show the same answer [1].
4. Stress that to calculate the volume of a box you multiply the three dimensions together [14].

STUDENTS DO

1. Watch the clip and follow the count of cubes per edge.
2. Copy the layer count and multiplication side by side.
3. Choral respond to edge counts and layer totals.

Say: We pack with no gaps to count volume. Then we multiply length times width times height. You will see both ways land on the same volume, in cubic units. Watch how the edge counts become the fraction multiplication.

Worked example: Prism 2 1/2 in by 2 in by 1 1/2 in packed with 1/2-in cubes. Step 1: cubes on edges = 5, 4, 3. Step 2: one layer = 5 x 4 = 20 tiny cubes. Step 3: 3 layers = 60 tiny cubes. Step 4: each tiny cube = 1/2 x 1/2 x 1/2 = 1/8 in3, so 60 x 1/8 = 60/8 = 7 1/2 in3. Step 5: multiply edges: 5/2 x 2 x 3/2 = 30/4 = 7 1/2 in3. Same answer [1].

Ask: If one layer holds 20 tiny cubes and there are 3 layers, how many tiny cubes in all? (60 tiny cubes, because volume can be number of cubes in one layer times number of layers [3].) · Why is the answer not 60 cubic inches? (60 is the count of 1/8-in3 cubes; 60 x 1/8 = 7 1/2 cubic inches [10].)

Check understanding: Whiteboards up: write cubes per edge for 2 1/2, 2, 1 1/2. Reteach counting halves if many write 2, 2, 1.

Support: Keep a whole-number model beside the fractional model and count cubes in one layer times number of layers aloud [3]. **Stretch:** Ask what would change if tiny cubes were 1/3-unit and how the multiplication would change.

Use: Slides 4-6, Video clip 2 (0:00–5:00)

17–40 min **Main - Half-Cube Pack Attack** 23 min · Groups

Find fractional volume by packing and prove it matches multiplication.

YOU DO

1. Hand out 1/2-inch cubes, small empty boxes, rulers, and recording sheet.
2. Assign each group a prism with halves, such as 2 1/2 by 2 by 1 1/2 in, and show Slide 8 task steps.
3. Circulate to check no gaps, correct edge counts, and both calculations.
4. Stop halfway to highlight one group where packing and multiplying matched [1].

STUDENTS DO

1. Pack the box with 1/2-inch cubes, count cubes per edge and per layer.
2. Record total tiny cubes and convert using 1/8 in3 per cube.
3. Multiply edge lengths and complete Worksheet Task B to show both match.

Say: Pack your box with 1/2-inch cubes with no gaps. Count how many fit on length, width, and height. Record layers, total tiny cubes, and then multiply the edge lengths to prove you get the same volume in cubic units.

Ask: How many 1/2-inch cubes fit along 2 and 1/2 inches, and how do you know? (Five, because two halves make 1, so 2 and 1/2 holds five halves.) · How does your cube count connect to l x w x h? (Edge counts 5 x 4 x 3 give 60 cubes; multiplying 5/2 x 2 x 3/2 gives the same space as 7 1/2 in3 [1][14].)

Check understanding: Collect recording sheets: total cubes, conversion, and multiplication must all show 7 1/2 in3. Pull any group with a mismatch to recount one edge together.

Support: Start with a whole-number box built out of unit cubes, such as by building the prism out of cubes [20], then move to halves. **Stretch:** Try a box with 1/3 or 1/4 inch edges and predict how many tiny cubes fit in one whole cube.

Use: Slide 8, Worksheet Task B

40–52 min **Apply - Both Formulas** 12 min · Pairs

Use V = l w h and V = B h with fractions for real box problems.

YOU DO

1. Show Slide 9: eight half-inch cubes fill one cubic inch because 2 x 2 x 2 = 8.
2. Model V = l w h and V = B h on Box A and Box B from Slide 10.
3. Have pairs solve Worksheet Task C using fractions for both formulas [1].
4. Remind that a box has a volume of length x width x height [6] and the volume V of a prism is V = B h where B is base area [3].

STUDENTS DO

1. Find base area, then multiply by height for Box B.
2. Multiply three fractions for Box A and compare.
3. Complete Worksheet Task C showing both V = l x w x h and V = base area x height work.

Say: You do not always need all three edges. If you know base area B, just do base area times height. For a rectangular prism the product of length and width is the base area [17]. Pick the faster formula, but answer in cubic units.

Worked example: Box A 2 1/2 x 2 x 1 1/2 in: V = l w h = 5/2 x 2 x 3/2 = 30/4 = 7 1/2 in3. Box B with base 5 in2 and height 1 1/2 in: V = B h = 5 x 3/2 = 15/2 = 7 1/2 in3. They hold the same. Any face can be the base if height is perpendicular to it [20].

Ask: When is V = B h faster than V = l w h? (When base area is already given, multiply once by height [3].) · Which holds more, Box A or Box B, and how do you know? (Neither; both are 7 1/2 in3, so they hold the same.)

Check understanding: Whiteboard vote for volume of Box B with work. If B x h errors appear, color the base and label B and h before retry.

Support: Give a card where only height is fractional and let students use a fraction chart for the one multiplication. **Stretch:** Give a missing height with known volume and ask students to work backward to find it.

Use: Slides 9-10, Worksheet Task C

52–60 min **Plenary** 8 min · Whole class

Check that packing, multiplying, and both formulas work for fractions.

YOU DO

1. Run Slide 11 true or false with whole-class votes and reasons.
2. Fix the false idea that V = B x h only works with whole numbers.
3. Return to success criteria and have students self-rate thumbs.
4. Give exit ticket on whiteboards.

STUDENTS DO

1. Vote true or false and justify with cubic units.
2. Correct the false statement aloud.
3. Answer exit ticket and self-check against success criteria.

Say: A 1/4-inch cube has volume 1/64 cubic inches because 1/4 x 1/4 x 1/4 = 1/64. V = B x h works for fractions too. Show me on your board that packing and multiplying are the same idea.

Ask: True or false: A 1/4-in cube has volume 1/64 in3? (True, 1/4 x 1/4 x 1/4 = 1/64.) · True or false: V = B x h only works with whole numbers? (False; it works for fractions too [1][3].)

Check understanding: Exit ticket whiteboards: all three must show cubic units and matching methods. Re-model one fraction multiplication for anyone missing it.

Support: Provide sentence starter: Packing gives ___ cubes, multiplying gives ___, so both give ___. **Stretch:** Ask students to write a new tricky volume card with a hidden mistake for another pair.

Use: Slide 11

Common mistakes

Mistake	How to spot it	How to fix it
Saying total tiny cubes equals cubic units, like 60 cubes = 60 in3	Answer far too big and missing conversion by 1/8	Multiply cube count by one tiny cube volume; eight half-inch cubes fill one cubic inch [10].
Adding edges instead of multiplying, or multiplying all numbers on a diagram	Work shows plus signs or volume without cubic units	Check against a box has volume length x width x height [6]; volume is number of cubes that fit inside [3].
Thinking V = B h only works for whole numbers	Student will not use base area when height is 1 1/2	Re-solve same prism both ways to show V = l w h and V = B h match for fractions [1][3].

Support

- Start with a whole-number box built out of unit cubes, such as by building the prism out of cubes [20], then move to halves.
- Use fraction multiplication chart and halves-only edges first, with base color-coded as B.
- Provide half-finished recording frame with edge labels and sentence starters.

Stretch

- Try a box with 1/3 or 1/4 inch edges and predict how many tiny cubes fit in one whole cube.
- Design a second box with half the volume but different fractional edges.
- Work backward from volume and base area to find a missing fractional height.

For the LSE or teaching assistant

Stay with the packing groups: help count cubes per edge, keep layers even, and prompt recording of both packing and multiplication so they can show the match.

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Check for understanding

Question	Answer
A box is 3 in by 1 1/2 in by 2 in. Find the volume using V = l w h.	9 in3, because 3 x 3/2 x 2 = 9 cubic units [10].
A prism has base area 6 in2 and height 2 1/2 in. Find the volume using V = B h.	15 in3, because 6 x 5/2 = 15 [3].
Why does packing with 1/2-in cubes give the same as multiplying edges?	Counts of 1/2-cubes per edge are numerators over 2; converting total by 1/8 equals the fraction product [1].

Homework

Find the volume of a storage box at home with one fractional edge using both V = l w h and V = B h.

Sources

1. Grade 6 » Geometry | Common Core State Standards Initiative · thecorestandards.org · https://www.thecorestandards.org/Math/Content/6/G/
2. SST Grade 6 Mathematics TEKS - Texas Education Agency · tea.texas.gov · https://tea.texas.gov/data-reports/staar/sst-6-math-assessed-curriculum.pdf
3. SURFACE AREA AND VOLUME - University of Houston · online.math.uh.edu · https://online.math.uh.edu/MiddleSchool/Modules/Module_3_Measurement/Content/SurfaceAreaandVolume-TEXT.pdf
4. Volume Of Rectangular And Triangular Prisms Worksheet · epm.cnc.stanford.edu · https://epm.cnc.stanford.edu/volume-of-rectangular-and-triangular-prisms-worksheet
5. Exploring Surface Area and Volume of Rectangular Prisms · serc.carleton.edu · https://serc.carleton.edu/msta/activities/31811.html
6. Surface Area and Volume - Andrews University · andrews.edu · https://www.andrews.edu/~calkins/math/webtexts/geom10.htm
7. Surface Area and Volume of Solids - Andrews University · andrews.edu · https://www.andrews.edu/~rwright/geometry/powerpoints/Geometry%2012%20Surface%20Area%20and%20Volume.pdf
8. Volume Of Rectangular Prism Worksheet - Great Research · epm.cnc.stanford.edu · https://epm.cnc.stanford.edu/volume-of-rectangular-prism-worksheet
9. Volume formula intuition (article) | Khan Academy · en.khanacademy.org · https://en.khanacademy.org/math/geometry-home/geometry-volume-surface-area/geometry-volume-rect-prism/a/volume-formula-intuition
10. Volume of rectangular prisms review - Khan Academy · khanacademy.org · https://www.khanacademy.org/math/grade-6-math-cambridge/x1977d7ef0ed7ac14:perimeter-area-and-volume/x1977d7ef0ed7ac14:volume/a/volume-of-rectangular-prisms-review
11. Surface area and volume of prisms - KS3 Maths - BBC Bitesize · bbc.co.uk · https://www.bbc.co.uk/bitesize/articles/zmkn3qt
12. Volume of rectangular prisms - Measurement and Geometry – GeoGeb… · geogebra.org · http://www.geogebra.org/m/v3xPu6JH
13. Flexi answers - How do you solve a rectangular prism? | CK-12 Foundation · ck12.org · https://www.ck12.org/flexi/math-grade-7/cross-sections-of-prisms/how-do-you-solve-a-rectangular-prism/
14. Volume of a rectangular prism (video) | Khan Academy · en.khanacademy.org · https://en.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-rect-prism/v/volume-of-a-rectangular-prism-or-box-examples
15. Volume of Rectangular Prisms (With Example) · superteacherworksheets.com · https://www.superteacherworksheets.com/printable-pdf-worksheets/geometry/43895-Volume-of-Rectangular-Prisms-(With-Example)
16. Volume of a Rectangular Prism Worksheets · mathworksheets4kids.com · https://www.mathworksheets4kids.com/volume-rectangular-prisms.php
17. Volume of a Rectangular Prism - Formula, Examples, & Diagrams - Math Monks · mathmonks.com · https://mathmonks.com/prism/volume-of-a-rectangular-prism
18. Grade 7 Mathematics - Exploring Surface Area and Volume of Cubes, Rectangular Prisms, and Cylinders - CAPS 123 · caps123.co.za · https://caps123.co.za/grade-7-mathematics-exploring-surface-area-and-volume-of-cubes-rectangular-prisms-and-cylinders/
19. Lesson: Volume of prisms | Oak National Academy · thenational.academy · https://www.thenational.academy/teachers/lessons/volume-of-prisms
20. Volume formulas review (article) | Khan Academy · math.khanacademy.org · https://math.khanacademy.org/math/geometry-home/geometry-volume-surface-area/geometry-volume-cones/a/volume-formulas-review
21. Volume of prisms KS3 | Y8 Maths | Lesson Resources · thenational.academy · https://www.thenational.academy/teachers/programmes/maths-secondary-ks3/units/perimeter-area-and-volume/lessons/volume-of-prisms
22. Volume of a Rectangular Prism - Steps, Examples & Questions · thirdspacelearning.com · https://thirdspacelearning.com/us/math-resources/topic-guides/geometry/volume-of-a-rectangular-prism/

23. rectangular prisms worksheets | Worksheetzone · worksheetzone.org · https://worksheetzone.org/worksheets/math/geometry/shapes/3d-shapes/rectangular-prisms